Ableitung von Umkehrfunktionen

Verfahren an einem Beispiel:

$$y = \arcsin x \Rightarrow (x = \sin y)$$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{\cos y} = \frac{1}{\cos(\arcsin x)}$$

$$\cos^2 u + \sin^2 u = 1$$

$$\Rightarrow \cos u = \sqrt{1-\sin^2 u}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\cos(\underbrace{\arcsin x}_{u})} = \frac{1}{\sqrt{1-\underbrace{\sin^2(\arcsin x)}_{x^2}}}$$

$$\Rightarrow \left\| \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} = (\arcsin x)' \right\|$$

Anderes Beispiel:

$$y = \operatorname{arctg} x \Rightarrow (x = \operatorname{tg} y)$$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{\frac{1}{\cos^2 y}} = \cos^2 y = \cos^2(\operatorname{arctg} x)$$

$$\boxed{\begin{aligned} &\operatorname{tg}^2 u = \frac{\sin^2 u}{\cos^2 u} = \frac{1-\cos^2 u}{\cos^2 u} \\ &\Rightarrow \cos^2 u(\operatorname{tg}^2 u + 1) = 1 \\ &\Rightarrow \cos^2 u = \frac{1}{1+\operatorname{tg}^2 u} \end{aligned}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{1+\operatorname{tg}^2(\operatorname{arctg} x)} = \frac{1}{1+x^2}$$

$$\Rightarrow \left\| (\operatorname{arctg} x)' = \frac{1}{1+x^2} \right\|$$

Weiteres Beispiel

$$y = \ln x \Rightarrow (x = e^y)$$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{e^y} = \frac{1}{e^{\ln x}} = \frac{1}{x}$$

$$\Rightarrow \left\| (\ln x)' = \frac{1}{x} \right\|$$

Weiteres Beispiel:

$$y = \operatorname{Arsinh} x \Rightarrow (x = \sinh y)$$

$$y' = \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{\cosh y} = \frac{1}{\cosh(\operatorname{Arsinh} x)}$$

$$\cosh^2 u - \sinh^2 u = 1$$

$$\Rightarrow \cosh u = \sqrt{1 + \sinh^2 u}$$

$$\Rightarrow \left\| y' = \frac{1}{\sqrt{1+x^2}} \right\| = (\operatorname{Arsinh} x)'$$